Neutrinospheres, resonant neutrino oscillations, and pulsar kicks

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Abstract

Pulsars are rapidly rotating neutron stars and are the outcome of the collapse of the core of a massive star with a mass of the order of or larger than eight solar masses. This process releases a huge gravitational energy of about 10^{53} erg, mainly in the form of neutrinos. During the collapse the density increases, and so does the magnetic field due to the trapping of the flux lines of the progenitor star by the high conductivity plasma. When the density reaches a value of around $10^{12}\,\mathrm{g\,cm^{-3}}$ neutrinos become trapped within the protoneutron star and a neutrinosphere, characterized inside by a diffusive transport of neutrinos and outside by a free streaming of neutrinos, is formed and lasts for a few seconds. Here we focus on the structure of the neutrinosphere, the resonant flavor conversion that can happen in its interior, and the neutrino flux anisotropies induced by this phenomena in the presence of a strong magnetic field. We present a detailed discussion in the context of the spherical Eddington model, which provides a simple but reasonable description of a static neutrino atmosphere, locally homogenous and isotropic. Energy and momentum are transported by neutrinos and antineutrinos flowing through an ideal gas of nonrelativistic, nondegenerate nucleons and relativistic, degenerate electrons and positrons. We examine the details of the asymmetric neutrino emission driven by active-sterile neutrino oscillations in the magnetized protoneutron star, and the possibility for this mechanism to explain the intrinsic large velocities of pulsars respect to nearby stars and associated supernova remnants.

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1 Introduction

The finding of pulsars in the sixties was one of the most astonishing discoveries in astrophysics[1]. Soon after their observation they were identified with neutron stars[2], whose existence had been theoretically predicted decades before. Neutron stars are the densest, most rapidly rotating, and most strongly magnetized objects in the Galaxy, features well understood by the standard physics of gravitational and elementary particle physics[3]. They are unique laboratories, which give us access to extreme conditions that are virtually impossible to obtain on Earth. Neutron stars are of particular interest for neutrino physics because of the key role that these particles play in the very first stages of their formation[4].

There is a remarkable characteristic of pulsars not satisfactorily understood: their large drift velocities with respect to nearby stars. The average value of pulsar velocities is in the range of $200 - 500 \, \mathrm{km \, s^{-1}}$, an order of magnitude larger than the mean velocity of ordinary stars in our Galaxy ($\sim 30 \, \mathrm{km \, s^{-1}}$)[5]. In addition, the distribution of these velocities is not Gaussian. It seems to have a bimodal structure[6], with a significant fraction ($\sim 15\%$) of the pulsar population having velocities higher than $1000 \, \mathrm{km \, s^{-1}}$ and up to a maximum of $1600 \, \mathrm{km \, s^{-1}}$.

Neutron stars are formed in type-II supernova explosions and, given the enormous energy liberated during these processes, it is quite natural to look for an explanation in terms of an impulse or *kick* received during their birth. This hypothesis is supported by recent analysis of individual pulsar motions and of associations between supernovae remnants and pulsars. It is also consistent with observed characteristics of binary systems with one or both constituents being a neutron star[7]. Despite this evidence, the physical origin of the kick is one of the most important unsolved puzzles in supernovae research. If the distribution is bimodal, then more than one mechanism could be responsible for the natal kicks. Two classes of mechanism have been considered[8]. One of them invokes an asymmetric mass ejection during the supernova explosion as the result of hydrodynamic perturbations caused by global density inhomogeneities prior to the core collapse[9] or convective instabilities in the neutrino-heated layer behind the shock[10]. Recoil speeds as high as $\sim 500 \, \mathrm{km \, s^{-1}}$ have been obtained with two-dimensional simulations. However, a recent three-dimensional calculation shows that even the most extreme asymmetric collapse does not produce final neutron star velocities above $200 \, \mathrm{km \, s^{-1}}$[11], rendering the situation unclear.

Only a small fraction of the gravitational binding energy ($E \simeq 10^{53} \, \mathrm{erg}$) liberated during a supernova explosion is required to account for the huge electromagnetic luminosity observed and the ejection of the envelope. More than 99% of the energy is released in the form of neutrinos, which are copiously produced when the iron core turns into a neutron core. A small asymmetry ($\sim 1\%$) in the momentum taken away by neutrinos should be enough to give the nascent pulsar an acceleration consistent with the measured velocities. The asymmetric neutrino emission induced by strong magnetic fields is the basis of the second class of kick mechanisms, and effects such as parity violation[12], asymmetric

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field distributions[13], and dark spots[14] have been considered in this context.

The density in the interior of a protoneutron star is so high, of the order of the nuclear density, that even neutrinos remain trapped, being emitted from an approximately well defined surface, the neutrinosphere. In a magnetized stellar plasma there are anisotropic contributions to the neutrino refraction index[15, 16] and, as a consequence, the resonant flavor transformations of neutrinos diffusing in different directions respect to the magnetic field occur at different depths within the protostar. This phenomenon can induce a momentum flux asymmetry and is the basis of the neutrino-driven kick mechanism proposed by Kusenko and Segrè (KS)[17, 18, 19]. With standard (active) neutrinos the mechanism works when the resonance region lies between the ν_e and ν_μ (or ν_τ) neutrinospheres, but requires an exceedingly high square mass difference, in conflict with the existing limits. A variant avoiding such a limitation can be implemented in terms of active-sterile neutrino oscillation[20, 21, 22].

The measurement of the Z-boson width at LEP, limiting to three the number of light neutrinos having weak interactions, does not preclude the existence of additional neutral fermions that do not fill the standard model gauge interactions. They have been invoked to explain several questions in astronomy and cosmology[23] and may becomes a necessity if the claimed LSND evidence for oscillations were confirmed. The range of the oscillation parameters (mass and mixing angle) required by the pulsar kick mechanism overlaps with the allowed region for sterile neutrinos being candidates to cosmological dark matter. Alternative kick mechanisms driven by neutrino oscillations have been proposed invoking the possible existence of transition magnetic moments[24], non orthonormality of the flavor neutrinos[25], violations of the equivalence principle[26], or off-resonant emission of sterile neutrinos[21].

In this article we review the explanation of the pulsar kick in terms of the resonant neutrino oscillations. In Section II, we outline the creation of a protoneutron star as a result of a supernova event, and the formation in its interior of a gas of trapped neutrinos. The momentum flux carried by neutrinos is analyzed in Section III. Section IV is devoted to the discussion of the neutrino oscillations in matter incorporating the effect of strong magnetic fields. The implication of this phenomenon as source of the asymmetry in the momentum flux transported by neutrinos emitted during the protoneutron star cooling is examined in Section V. Numerical results are obtained in Section VI within the context of the Eddington model, where the kick effect is explicitly evaluated. In the final section we summarize the outstanding features of the mechanism.

2 Stellar collapse, protoneutron stars, and neutrinospheres

Neutron stars are extremely compact objects born in type II supernova explosions[27], as the aftermath of the gravitational collapse of a massive star ($M \gtrsim 8M_\odot$) when it exhausts the sources of thermonuclear energy and its iron core reaches the

Chandrasekhar limit, $M_{Ch} = 1.44 \, M_\odot$. The process is triggered by the photodissociation of iron nuclei, $\gamma + {}^{56}\text{Fe} \rightarrow 13\,{}^{4}\text{He} + 4n - 124,4\,\text{MeV}$, which consumes energy reducing the pressure of the degenerated electron gas. When the collapse proceeds, pressure support decreases because of the electron capture by nuclei. Electron neutrinos are copiously produced through the reaction $p + e \rightarrow n + \nu_e$ and initially they freely escape from the core, but as density increases the medium becomes less transparent for them. The inner core ($M \approx 0.5 - 0.8 \, M_\odot$) collapses in a homologous way at a subsonic velocity proportional to the radius ($v/r = 400 - 700 \, \text{s}^{-1}$), with the density and temperature profiles remaining similar and only the scale changing with time[28]. The external shells collapse at a supersonic velocity of the order of free fall velocity ($v \propto 1/\sqrt{r}$).

During the first part of the collapse the main source of opacity for neutrinos is the coherent scattering by heavy nuclei, mediated by the neutral current weak interaction. The cross section for this reaction is proportional to A^2, where A is the number of nucleons in the nuclei, and is very effective in trapping neutrinos[29]. If λ_ν is the mean free path of neutrinos, then the time for them to diffuse out of the core, estimated from a random walk through a distance of the order of the core radius R, is $t_{diff} \sim 3R^2/\lambda_\nu$. On the other hand, the core dynamical time scale is of the order of the free fall time $t_c \sim (G\rho)^{-1/2}$. At a certain density, of about $10^{12} \, \text{g cm}^3$ for 10 MeV neutrinos, both times become comparable and neutrinos are trapped[30]. β-equilibrium between e and ν_e is set up and electron neutrinos also become degenerated. In this way, the collapsing core is the only place, besides the early universe, where neutrinos are in thermal equilibrium. During the collapse the lepton fraction $Y_L = Y_e + Y_{\nu_e}$ is kept almost constant near the value of the electron fraction at the beginning of the trapping ($Y_e \approx 0.35$), with only $\sim 1/4$ corresponding to neutrinos[4].

The degenerate gas of trapped neutrinos diffuses towards the exterior, reaching regions of decreasing density where the collisions are less frequent, until they finally decouple from the star. Although this process is continuous, it is convenient to introduce an effective emission surface called the neutrinosphere, characterized as the surface where the optical depth becomes approximately one. More accurately, the radius of the neutrinosphere R_{ν_ℓ} is defined by the condition

$$\int_{R_{\nu_\ell}}^{\infty} \frac{dr}{\lambda_{\nu_\ell}} \sim \frac{2}{3} \, . \tag{1}$$

The medium is considered opaque for neutrinos inside the neutrinosphere and transparent outside. We can also define the energy sphere, where the energy exchanging reactions freeze out while energy conserving collisions may still be important.

The collapse stops suddenly when, after a fraction of seconds, densities of around $3 \times 10^{14} \, \text{g cm}^{-3}$, exceeding nuclear density, are reached[31]. Then, a shock wave is built up near the interface of the halted inner core and the supersonically free falling outer part. The shock wave propagates outwards and eventually ejects the stellar mantle of the star. At its passage through the neu-

trinosphere nuclei are dissociated decreasing the neutrino opacity and causing a recession of the neutrinosphere. The ν_e produced by the reaction $p + e \rightarrow n + \nu_e$ can escape freely building up a short (< 10 msec) ν_e burst, called the "neutronization burst" or "deleptonization burst". The star remnant gravitationally decouples from the expanding ejecta and a protoneutron star has been formed[32].

The neutrinos continue trapped due to the interaction with the residual particles of the medium, mainly nucleons, electrons, and neutrinos. For ν_e the dominant reactions are the β and β-inverse, which mantain them in local thermodynamical equilibrium[33]. In this case, the distinction between neutrinosphere and energy sphere is not crucial. The electron neutrinos also interact with nucleons through the neutral current, but the cross sections for these reactions are smaller. Although the energy dependence of the process involving ν_e and $\bar{\nu}_e$ is the same, in a protoneutron star the number of neutrons is larger than the number of protons, and therefore the ν_e suffer more collisions than the $\bar{\nu}_e$. As a consequence, the neutrinosphere of the ν_e is a bit further out than the one of the $\bar{\nu}_e$, and the mean energy per emitted particle is smaller. After the achievement of β equilibrium, the total entropy is conserved and the collapse becomes adiabatic.

In the case of the nonelectron (active) neutrinos and antineutrinos, the absence of muons and taus does not allow the establishment of chemical equilibrium with the stellar plasma. Therefore, they do not transport leptonic number outside the star and their chemical potentials vanish. Their interactions with the background are the same and no distinction needs to be made between ν_μ and ν_τ. Deeper in the protostar the main interactions for these neutrinos are pair processes such as $\mathcal{N}\mathcal{N}' \rightleftarrows \mathcal{N}\mathcal{N}'\nu_{\mu,\tau}\bar{\nu}_{\mu,\tau}, e^-e^+ \rightleftarrows \nu_{\mu,\tau}\bar{\nu}_{\mu,\tau}$, etc, where $\mathcal{N}$ denotes a nucleon, and the scattering reactions with nucleons and electrons. The scattering on electrons are less frequent, but the energy transferred is greater. This process dominates the energy exchange up to the energy sphere where it freezes out. Beyond this radius, muon and tau neutrinos still scatter off neutrons until they reach the neutrinosphere. The cross section for $\mathcal{N}\nu_{\mu,\tau} \rightleftarrows \mathcal{N}\nu_{\mu,\tau}$ is smaller than the one of the β reaction, and hence the radius of the muon (tau) neutrinosphere $R_{\nu_{\mu,\tau}}$ is shorter than the radius of the electron neutrinosphere R_{ν_e}. The protoneutron star contracts, cools, and deleptonizes due to the emission of neutrinos and antineutrinos of all the flavors in a scale of time of the order of 10 sec. The released energy is equally shared by all neutrino and antineutrino species. This phase is known as the Kelving-Helmholtz phase and ends up in the formation of a neutron star. During this time the radius of the protostar varies from around 30 to 10 km, while the mass remains of the order of 1.4 $M_\odot$. After approximately 1 minute the neutrino mean free path becomes comparable to the stellar radius and neutrino luminosity decreases quickly.

Another important property predicted for neutron stars is that they should have an extremely strong magnetic field. Since the conductivity of the iron core is very large, the magnetic flux is essentially trapped and it is conserved as it collapses to form a neutron star[34]. Adopting for the initial magnetic field the typical values observed in some white dwarfs, from the flux conservation one can easily estimate that the surface field of a protoneutron star has to be in the range

of $10^{12} - 10^{13}$ G. This is supported by evidence inferred from measurements of the spin-down rates and observations of the X-ray spectra features[35]. Magnetic fields of the order of 10^{15} G or stronger can be generated by dynamo processes, and in the central regions intensities up to 10^{18} G have been considered[36].

3   Momentum flux

In a protoneutron star the momentum flux from the core to the external regions is dominated by neutrinos, which provide the most efficient mechanism to release the energy generated during stellar collapse. As explained in the previous section, matter densities are so high that neutrinos are trapped forming a degenerated gas at quasi-equilibrium diffusing radially. The statistical description of this gas is made in terms of a distribution function $f_{\nu_\ell}(\mathbf{x}, \mathbf{k}, t)$. If the action of external fields is negligible, then the neutrino momentum between two consecutive collisions remains constant. The change of the distribution function is due to neutrino collisions with the particles in the medium and is described by the Boltzmann equation

$$\frac{df_{\nu_\ell}}{dt} = C(f_{\nu_\ell}), \tag{2}$$

where $C(f_{\nu_\ell})$ is the collision integral and

$$\frac{df_{\nu_\ell}}{dt} = \frac{\partial f_{\nu_\ell}}{\partial t} + \hat{\mathbf{k}} \cdot \nabla f_{\nu_\ell}. \tag{3}$$

When the regime is close to equilibrium we can approximate the collision integral as follows

$$C(f_{\nu_\ell}) \simeq -\frac{f_{\nu_\ell} - f_{\nu_\ell}^{eq}}{\tau_{\nu_\ell}}, \tag{4}$$

where $\tau_{\nu_\ell} \cong \lambda_{\nu_\ell}$ is the average time between collisions and

$$f_{\nu_\ell}^{eq}(k) = \frac{1}{1 + e^{(k-\mu_{\nu_\ell})/T}} \tag{5}$$

is the Fermi-Dirac distribution function for the gas of relativistic neutrinos ($E_{\nu_\ell} \cong k = |\mathbf{k}|$) with chemical potential μ_{ν_ℓ} and temperature T. In the static situation ($\partial f_{\nu_\ell}/\partial t = 0$), Eq. (2) with $C(f_{\nu_\ell})$ given by Eq. (4) can be solved iteratively. Proceeding in this way we obtain

$$f_{\nu_\ell} \simeq f_{\nu_\ell}^{eq} - \lambda_{\nu_\ell} \hat{\mathbf{k}} \cdot \nabla f_{\nu_\ell}^{eq}, \tag{6}$$

which constitutes the diffusion approximation for the distribution function. The second term in this expression gives the correction to equilibrium and allows the transport of the energy and momentum towards the exterior of the protostar.

Once the distribution function is known the neutrino energy-momentum tensor can be computed as

$$T_{\nu_\ell}^{\mu\nu} = \int \frac{d^3 k}{(2\pi)^3} \frac{k^\mu k^\nu}{k^0} f_{\nu_l}. \tag{7}$$

From this formula, for the energy density we have

$$U_{\nu_l} = T_{\nu_l}^{00} = \frac{1}{(2\pi)^3} \int k^3 dk \sin\theta' d\theta' d\phi' f_{\nu_l} = \frac{F_3[\eta_{\nu_l}]}{2\pi^2} T^4 \,, \tag{8}$$

where $\eta_{\nu_l} = \mu_{\nu_l}/T$ is the degeneracy parameter and $F_3[\eta_{\nu_l}]$ is a Fermi integral

$$F_n[\eta_{\nu_l}] = \int_0^\infty dx\, x^n [1 + \exp(x - \eta_{\nu_l})]^{-1}. \tag{9}$$

At each point within the protostar we choose a local frame (x', y', z') with $\hat{\mathbf{r}} = \hat{\mathbf{z}}'$, such that $\hat{\mathbf{k}} = \cos\theta' \hat{\mathbf{z}}' + \sin\theta' \cos\phi' \hat{\mathbf{x}}' + \sin\theta' \sin\phi' \hat{\mathbf{y}}'$.

The stress tensor can be expressed in terms of the energy density as

$$T_{\nu_l}^{ij} = \delta^{ij} \frac{U_{\nu_l}}{3}. \tag{10}$$

The equilibrium component of the distribution function is isotropic and gives a null contribution to the momentum flux ($F_{\nu_l}^i = T_{\nu_l}^{0i}$), which is generated exclusively by the diffusive term:

$$\mathbf{F}_{\nu_\ell} = \frac{1}{(2\pi)^3} \int k^2 dk \sin\theta' d\theta' d\phi' \, \mathbf{k} f_{\nu_\ell} = -\frac{1}{6\pi^2} \int_0^\infty k^3 \lambda_{\nu_\ell} \nabla f_{\nu_\ell}^{eq} dk \,. \tag{11}$$

To complete the calculation of $\mathbf{F}_{\nu_\ell}$ we need the mean free paths of the neutrinos or equivalently, the opacities κ_{ν_ℓ}.

The neutral current reactions equally affect all flavors. Since the energies available in the system are much smaller than the mass of the τ lepton, the ν_τ does not participate in the charge-current interactions. For the same reason, and for simplicity, we also ignore the presence of muons. Accordingly, the main sources of neutrino opacities are

$$\nu_\ell + n \rightleftarrows \nu_\ell + n \,, \tag{12}$$

$$\nu_\ell + p \rightleftarrows \nu_\ell + p \,, \tag{13}$$

$$\nu_e + n \rightleftarrows e^- + p \,. \tag{14}$$

In terms of the cross sections for these reactions the $\lambda_{\nu_\ell}^{-1}$ are given by

$$\lambda_{\nu_e}^{-1} = \lambda_{\nu_{\mu,\tau}}^{-1} + N_n \sigma_{abs}, \tag{15}$$

$$\lambda_{\nu_{\mu,\tau}}^{-1} = N_n \sigma_n + N_p \sigma_p \,, \tag{16}$$

where $N_{n,p}$ are the neutron and proton number densities and

$$\sigma_n = \frac{G_F^2}{4\pi} \left(1 + 3g_A^2\right) k^2 \,, \tag{17}$$

$$\sigma_p = \sigma_n \left[1 - \frac{8\sin^2\theta_W}{1 + 3g_A^2} \left(1 - 2\sin^2\theta_W\right) \right] \,, \tag{18}$$

$$\sigma_{abs} = 4\,\sigma_n \,. \tag{19}$$

Here, $\sin^2\theta_W \simeq 0.23$ and $g_A \simeq 1.26$ is the renormalization for the axial-vector current of the nucleons. In Eq. (19) we neglected contributions of order $(m_n - m_p)/k$, where m_n and m_p are the neutron and proton mass, respectively.

From the previous formulae we immediately see that

$$\lambda_{\nu_\ell}^{-1} = \kappa_{\nu_\ell}\rho = \varkappa_{\nu_\ell} k^2 \rho \,, \tag{20}$$

where the coefficients $\varkappa_{\nu_\ell}$ are constants. By making $m_n \cong m_p$ and taking the typical values $Y_n \approx 0.9$ and $Y_p \approx 0.1$ for the nucleon fractions $Y_{n,p} = N_{n,p}/(N_n + N_p)$, we get

$$\varkappa_{\nu_e} = 3 \times 10^{-25} \, \text{MeV}^{-5}, \tag{21}$$

$$\varkappa_{\nu_{\mu,\tau}} = 0.7 \times 10^{-25} \, \text{MeV}^{-5}. \tag{22}$$

When Eq. (20) is substituted in the integrand of Eq. (11), the momentum flux transported by neutrinos diffusing from the core to the exterior of the protostar becomes

$$\mathbf{F}_{\nu_\ell} = -\frac{1}{6\pi^2}\frac{1}{\varkappa_{\nu_\ell}\rho}\nabla\left(F_1[\eta_{\nu_\ell}]\,T^2\right). \tag{23}$$

In what follows we assume that the neutrino chemical potentials are negligible, and using $F_1[0] = \pi^2/12$ and $F_3[0] = 7\pi^4/120$, from Eqs. (8) and (23), we obtain

$$U_{\nu_\ell} = \frac{7\pi^2}{240}\,T^4 \,, \tag{24}$$

$$\mathbf{F}_{\nu_\ell} = -\frac{1}{72\varkappa_{\nu_\ell}\rho}\,\nabla T^2 \,. \tag{25}$$

The last results will be used in Section 6, when we calculate the asymmetric neutrino emission in the context of a specific model for the protoneutron star.

4 Neutrino oscillations in a magnetized medium

A notable consequence of neutrino mixing is the phenomena of neutrino oscillations established five decades ago by Pontecorvo[37, 38]. It corresponds to the periodic variation in the flavor content of a neutrino beam as it evolves from the production point. In the last years compelling evidence of neutrino oscillations has been obtained in experiments with atmospheric, solar, accelerator, and reactor neutrinos[39, 40]. The most plausible interpretation for these new phenomena is that neutrinos are massive particles and the weak-interaction states $|\nu_\ell\rangle$ $(\ell = e, \mu, \tau)$ are linear combinations of the mass eigenstates $|\nu_j\rangle$ $(j = 1, 2, 3)$, with masses m_j:

$$|\nu_\ell\rangle = U_{\ell j}\,|\nu_j\rangle \,, \tag{26}$$

where $U_{\ell j}$ are the elements of a unitary mixing matrix, the analogue of the CKM matrix in the quark sector. Consider a beam of ν_α created at a certain point with a 3-momentum $\mathbf{k}$. When the state given in Eq. (26) evolves, each

of the mass eigenstates acquires a different phase $e^{-iE_j t}$, with $E_j = \sqrt{m_j^2 + k^2}$ ($k = |\mathbf{k}|$). As a consequence, at a certain distance $r \cong t$ from the source the neutrino beam will not correspond to pure ν_ℓ but turns partially into other flavors.

The pattern of neutrino oscillations is modified when neutrinos propagate through a material medium. The basic reason is that the energy-momentum relation of an active neutrino is affected by its coherent interactions with the particles that constitute the medium. These modifications are described in terms of a potential energy V_ℓ, which is related to the real part of the refraction index n_ℓ according to $\mathrm{Re}\, n_\ell = 1 - V_\ell/k$ and can be calculated from the background contributions to the neutrino self-energy[41]. If, like in normal matter, electrons but not heavier leptons are present in the background, then only ν_e scatter through charge current interactions, while all the flavors interact via the neutral current. Therefore, the refraction index differs for neutrinos of different flavors and they will acquire distinct phases when propagating with the same momentum through the medium. This fact can have impressive consequences in the case of mixed neutrinos and, under favorable conditions, flavor transformations are significantly enhanced in a medium with a varying density even when mixing in vacuum is small. This is the essence of the MSW mechanism[42, 38] whose large mixing realization provides the solution to the solar neutrino problem[40, 43].

Besides the modifications of the dispersion relation, neutrinos acquire in matter an effective coupling to the electromagnetic field by means of their weak interaction with the background particles[15]. As a result, in the presence of an external magnetic field there are additional contributions to the neutrino refraction index and flavor transformations are affected, but in a way that preserves quirality[44], contrary to what happens for neutrino oscillations driven by transition magnetic moments[45]. In what follows, we will discuss neutrino oscillations in a magnetized medium for the simplest situation of mixing between two neutrino species, that is, ν_ℓ and another active neutrino $\nu_{\ell'}$ or a hypothetical sterile neutrino ν_s. In any case we can write

$$\mathbb{U} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{27}$$

where θ is the vacuum mixing angle, a parameter to be fixed from the experimental results.

The Dirac wave function for relativistic (mixed) neutrinos with momentum $\mathbf{k}$ propagating in matter in the presence of a static uniform magnetic field is well approximated as follows[44]:

$$\psi \cong e^{i\mathbf{k}\cdot\mathbf{x}} \begin{pmatrix} 0 \\ \phi_- \end{pmatrix} \chi(t), \tag{28}$$

where we use the Weyl representation, ϕ_- is the Pauli spinor of negative helicity ($\vec{\sigma} \cdot \hat{\varkappa}\, \phi_- = -\phi_-$) and $\chi(t)$ is a flavor-space spinor that satisfies

$$i\frac{d\chi}{dt} = \mathbb{H}_B \chi. \tag{29}$$

The Hamiltonian $\mathbb{H}_B$ is a 2×2 matrix that, in the basis of the flavor states, takes the form

$$\mathbb{H}_B = k + \frac{\mathbb{M}^2}{2k} + \mathbb{V}, \tag{30}$$

with

$$\mathbb{M} = \mathbb{U} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \mathbb{U}^\dagger \tag{31}$$

and $\mathbb{V} = \mathrm{diag}(V_\ell, V_{\ell',s})$.

The effective potential for active neutrinos can be written as[15, 47]

$$V_\ell = b_\ell - c_\ell \, e \, \hat{\mathbf{k}} \cdot \mathbf{B} \,, \tag{32}$$

where $e > 0$ is the proton charge. The coefficients b_ℓ and c_ℓ depend on the properties of the thermal background. To order G_F the expression for b_ℓ is independent of B, but this is not true in general for c_ℓ. In our analysis we will work within the weak-field limit, i.e., $B \ll \mu_e^2/2e$[46], and ignore any possible dependence of this coefficient on the magnetic field. The effect of strong magnetic fields on the neutrino propagation in a medium have been considered by some authors[47]. For (active) antineutrinos the potentials change in sign, i.e., $\bar{V}_\ell = -V_\ell$, while in the case of sterile neutrinos, since they do not interact with the background, we have

$$V_s = \bar{V}_s = 0 \,. \tag{33}$$

Eq. (29) is the extension of the MSW equation to the situation we are considering, and was first derived in Ref. [44]. The components $\chi_e(t)$ and $\chi_{\mu,s}(t)$ of $\chi(t)$ give the amplitude to find the neutrino in the corresponding flavor state. For a uniform background $\chi(t) = \sum\limits_{i=1,2} (\varsigma_i^\dagger \, \chi(0))\varsigma_i e^{i\omega_i t}$, where $\omega_i(p)$ are the energies of the neutrino modes in the medium and the vectors $\varsigma_{1,2}$ are the solutions of the eigenvalue condition $\mathbb{H}_B \varsigma_i = \omega_i \varsigma_i$. For oscillations the relevant quantity is

$$\omega_2 - \omega_1 = \frac{1}{2k} \sqrt{\left(\Delta m^2 \cos 2\theta - 2k\mathcal{V}\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2} \,, \tag{34}$$

where $\mathcal{V}(r) = V_\ell(r) - V_{\ell',s}(r)$ and $\Delta m^2 \equiv m_2^2 - m_1^2$. When the medium is inhomogenous, along the neutrino path ($r \cong t$) the elements of $\mathbb{V}$ are functions of t, and $\chi(t)$ has to be determined by solving Eq. (29) with $\mathbb{H}_B$ a time dependent matrix. At each point r, $\mathbb{H}_B$ can be diagonalized by the unitary transformation

$$\mathbb{U}_m(r) = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \,, \tag{35}$$

with the mixing angle in matter $\theta_m(r)$ given by

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2k\mathcal{V}} \,. \tag{36}$$

A similar expression is valid in the case of antineutrinos with $\Delta m^2 \cos 2\theta + 2k\mathcal{V}$ in the denominator. In general $\mathcal{V}$ and consequently the mixing angle in

matter depends on the magnetic field. As a function of $\mathcal{V}$, $\sin^2 2\theta_m$ exhibits the characteristic form of a Breit-Wigner resonance. For neutrinos (antineutrinos) with momentum $\mathbf{k}$ the resonance condition ($\sin^2 2\theta_m = 1$) is

$$\mathcal{V}(r_R) = \pm \frac{\Delta m^2}{2k} \cos 2\theta, \qquad (37)$$

where $r_R = r_R(\mathbf{k})$ and the sign $+$ $(-)$ corresponds to neutrinos (antineutrinos). Notice that for active-active oscillations the neutral current contributions to $\mathcal{V} = V_\ell - V_{\ell'}$ cancel since they are the same for both flavors. This is not true in the case of active-sterile oscillations where $\mathcal{V} = V_\ell$, and such contributions play a relevant role in the resonant transformations for $\nu_\ell \longleftrightarrow \nu_s$ (or $\bar{\nu}_\ell \longleftrightarrow \bar{\nu}_s$). The efficiency of the flavor transformation depends on the adiabaticity of the process. The conversion will be adiabatic when the oscillation length at the resonance is smaller than the width of the enhancement region. This imposes the condition

$$\left| \frac{d\mathcal{V}}{dr} \right|_{r_R} < \left(\frac{\Delta m^2}{2k} \sin 2\theta \right)^2 . \qquad (38)$$

The dispersion relation of neutrinos in a magnetized plasma made of electrons, protons, and nucleons have been calculated in Ref. [16], including the contributions due to the anomalous magnetic moment coupling of the nucleons to the photon. From the results given there we have

$$b_e = \sqrt{2} G_F \left[N_e + \left(\frac{1}{2} - 2\sin^2 \theta_W \right) (N_p - N_e) - \frac{1}{2} N_n \right] + \tilde{b}_e, \qquad (39)$$

$$b_{\mu,\tau} = \sqrt{2} G_F \left[\left(\frac{1}{2} - 2\sin^2 \theta_W \right) (N_p - N_e) - \frac{1}{2} N_n \right] + \tilde{b}_{\mu,\tau}, \qquad (40)$$

$$c_e = 2\sqrt{2} G_F \left[g_A \left(1 + 2m_p \frac{\kappa_p}{e} \right) C_p - g_A 2 m_n \frac{\kappa_n}{e} C_n - C_e \right], \qquad (41)$$

$$c_{\mu,\tau} = 2\sqrt{2} G_F \left[g_A \left(1 + 2m_p \frac{\kappa_p}{e} \right) C_p - g_A 2 m_n \frac{\kappa_n}{e} C_n + C_e \right], \qquad (42)$$

where $\kappa_{n,p}$ are the anomalous part of the nucleon magnetic moments given by

$$\kappa_n = -1.91 \frac{e}{2m_n} , \qquad \kappa_p = 1.79 \frac{e}{2m_p} , \qquad (43)$$

and

$$\tilde{b}_\ell = \sqrt{2} G_F \left[N_{\nu_\ell} - N_{\bar{\nu}_\ell} + \sum_{\ell'} \left(N_{\nu_{\ell'}} - N_{\bar{\nu}_{\ell'}} \right) \right] , \qquad (44)$$

$$C_\flat = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E} \frac{d}{dE} f_\flat \qquad (\flat = e, n, p) . \qquad (45)$$

In these equations N_e and $N_{\nu_\ell} (N_{\bar{\nu}_\ell})$ are the number densities of electrons and neutrinos (antineutrinos) of flavor ℓ, respectively and $f_\flat$ are the distribution functions of the electrons, protons, and neutrons.

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We model the atmosphere of a protoneutron star as a neutrino gas diffusing through a plasma made of relativistic degenerate electrons and classical nonrelativistic nucleons. In such conditions $C_e = -\mu_e/8\pi^2$ and $C_{n,p} = -N_{n,p}/8Tm_{n,p}$[16, 46], where T is the background temperature and $\mu_e = \left(3\pi^2 N_e\right)^{1/3}$ is the electron chemical potential. The neutrino fractions are much smaller than the electron fraction $Y_e = N_e/(N_n + N_p) \sim 0.1$ and, in the context of our analysis, the contributions to b_{ν_ℓ} coming from the neutrino-neutrino interactions (denoted by $\tilde{b}_\ell$) can be neglected. With this approximation in mind, putting $m_p \simeq m_n$, and taking into account that by electrical neutrality $N_e = N_p$, the coefficients in the effective potentials become:

$$b_e \simeq -\frac{G_F}{\sqrt{2}}\,(1-3Y_e)\,\frac{\rho}{m_n}\,, \tag{46}$$

$$b_{\mu,\tau} \simeq -\frac{G_F}{\sqrt{2}}\,(1-Y_e)\,\frac{\rho}{m_n}\,, \tag{47}$$

$$c_e \simeq -\sqrt{2}\,G_F\left(\frac{3+Y_e}{5Tm_n^2}\,\rho - \frac{\mu_e}{4\pi^2}\right), \tag{48}$$

$$c_{\mu,\tau} \simeq -\sqrt{2}\,G_F\left(\frac{3+Y_e}{5Tm_n^2}\,\rho + \frac{\mu_e}{4\pi^2}\right). \tag{49}$$

It should be noticed that in Ref. [20] c_e vanishes because all the components of the stellar plasma are assumed to be degenerated, and the contributions due to the anomalous magnetic moment of the nucleons are not included.

We now use the effective potentials V_ℓ with b_ℓ and c_ℓ given by Eqs. (46)-(49) to examine the magnetic field effect on the resonant neutrino oscillations within a protoneutron star. In a linear approximation, we can write

$$r_R(\mathbf{k}) = r_o(k) + \delta(k)\cos\alpha\,, \tag{50}$$

where r_o and δ are ordinary functions of the magnitude of the neutrino momentum and α is the angle between $\mathbf{k}$ and $\mathbf{B}$. For a certain value of k, Eq. (50) determines a spherical shell limited by the spheres of radii $r_o(k) \pm \delta(k)$, where the resonance condition is verified for neutrinos (or antineutrinos) moving in different directions with respect to the magnetic field. The quantity r_o corresponds to the radius of the resonance sphere when $B = 0$ and in the case of oscillations between ν_e and ν_μ (or ν_τ) is given by

$$G_F\sqrt{2}\,Y_e\frac{\rho}{m_n}\bigg|_{r_o} = \pm\frac{\Delta m^2}{2k}\cos 2\theta\,. \tag{51}$$

The left-hand side corresponds to the difference $b_e - b_{\mu,\tau}$ evaluated at $r = r_o$ and is positive definite. Therefore, assuming $\Delta m^2 \cos 2\theta > 0$, the above condition can be satisfied for $\nu_e \longleftrightarrow \nu_{\mu,\tau}$ but not for $\bar{\nu}_e \longleftrightarrow \bar{\nu}_{\mu,\tau}$. To be effective as a kick mechanism the resonance transformations of active neutrinos have to take place in the region between the electron and muon (tau) neutrinospheres. In this case, the ν_e are trapped by the medium, but the $\nu_{\mu,\tau}$ produced this way are

above their neutrinosphere and, when moving outside, can freely escape from the star. In the presence of a magnetic field, the emission points for $\nu_{\mu,\tau}$ having the same k but different directions are not at the same radius, which originates an asymmetry in the momentum they carried away.

Replacing k in Eq. (51) by the thermal average $\langle k \rangle \simeq 3.15\,T$ and taking into account that $\rho \sim 10^{11}\,\mathrm{g\,cm}^3$ and $T \sim 4\,\mathrm{MeV}$ at R_{ν_e}, the requirement of having the resonance within the electron neutrinosphere implies $\Delta m^2 \cos 2\theta \gtrsim 2 \times 10^4\,\mathrm{eV}^2$, which is excluded by the experimental results on neutrino oscillations and the cosmological limits. For this reason, in what follows we concentrate only on the active-sterile oscillations. In this case, the resonance condition when $B = 0$ takes the form

$$b_\ell(r_{o_\ell}) = \mp \frac{\Delta m^2}{2k} \cos 2\theta \,, \tag{52}$$

for $\nu_\ell \longleftrightarrow \nu_s$ ($\bar\nu_\ell \longleftrightarrow \bar\nu_s$). In the static model we use for the protoneutron star, a good approximation is to take $Y_e < 1/3$. Therefore, if $\Delta m^2 \cos 2\theta > 0$, the resonant condition is verified only by antineutrinos and from now on we restrict ourselves to this situation.

To determine the quantity δ in Eq. (50) we substitute V_ℓ as given by Eq. (32) in Eq. (37) and expand b_ℓ up to first order in δ_ℓ. Proceeding in this way we get

$$\delta_\ell(k) = \mathcal{D}_\ell(k)\,eB \,, \tag{53}$$

with

$$\mathcal{D}_\ell(k) = \left. \frac{1}{h_{b_\ell}^{-1}} \frac{c_\ell}{b_\ell} \right|_{r_{o_\ell}} \,, \tag{54}$$

where we have defined $h_g^{-1} \equiv \frac{d}{dr} \ln g$ for any function $g(r)$. Using Eqs. (46)-(49) we find explicitly

$$\mathcal{D}_e = \left. \frac{2}{(1 - 3Y_e)\,h_\rho^{-1} - 3Y_e h_{Y_e}^{-1}} \left(\frac{3 + Y_e}{5Tm_n} - \frac{m_n \mu_e}{4\pi^2 \rho} \right) \right|_{r_{o_e}} \,, \tag{55}$$

$$\mathcal{D}_{\mu,\tau} = \left. \frac{2}{(1 - Y_e)\,h_\rho^{-1} - Y_e h_{Y_e}^{-1}} \left(\frac{3 + Y_e}{5Tm_n} + \frac{m_n \mu_e}{4\pi^2 \rho} \right) \right|_{r_{o_{\mu,\tau}}} \,. \tag{56}$$

Finally from Eq. (38), discarding terms proportional to the magnetic field, the following restriction to the vacuum mixing angle results

$$\tan^2 2\theta > \left. \left| \frac{h_{b_\ell}^{-1}}{b_\ell} \right| \right|_{r_{o_\ell}} \,, \tag{57}$$

which guarantees that the evolution will be adiabatic and an almost complete flavor transformation for small values of θ.

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5   Neutrino momentum asymmetry

If the temperature and density profiles are isotropic, then the momentum flux in the protoneutron star atmosphere is radial (see Eq. (25)). As a consequence, in most of the works on the kick mechanism driven by matter neutrino oscillations the resonance condition was evaluated assuming that neutrinos move outside with a momentum $\langle k \rangle = (7\pi^4/180\,\zeta(3))\,T \simeq 3.15\,T$ pointing in the radial direction. In presence of a magnetic field this approach leads to the concept of a deformed resonance surface, which acts as an effective neutrinosphere. However, at each point within the neutrinosphere there are neutrinos with momentum $\mathbf{k}$ pointing in every directions and the resonance condition as given by Eq. (37) actually defines a spherical surface for each value of $\cos\alpha$. This surface acts as a source of sterile neutrinos produced through the resonant $\bar{\nu}_\ell$ transformations, which move in the $\hat{\mathbf{k}}$ direction. Therefore, as was mentioned above, for a given k the resonant transformations take place within a spherical shell, and not on a deformed spherical surface. A more careful calculation of the pulsar kick along this line has been carried out in Ref. [22] and here we follow the same approach.

Let us consider the situation where the resonance shell is totally contained within the $\bar{\nu}_\ell$ neutrinosphere. The sterile neutrinos produced there that move out freely escape from the protostar, but those directed toward the inside cross the resonance surface again and reconvert into $\bar{\nu}_\ell$ which, being within their own neutrinosphere, are thermalized. Consequently, only those $\bar{\nu}_s$ going outward can leave the star and the resonance surfaces for different momentum directions behave as effective emission semispheres of sterile antineutrinos. For $\mathbf{k}$ pointing in opposite directions the radii of the respective semispheres are $r_R^\pm = r_o(k) \pm \delta(k)\,|\cos\alpha|$, and therefore they have different areas that generate a difference in the momentum carried away by the sterile neutrinos leaving the star in opposite directions. There also exists a temperature variation within the resonance shell such that the temperatures of the emission semispheres for $\bar{\nu}_s$ going outside in opposite directions are different $(T(r_R^-) > T(r_R^+))$. The variations in the area and the temperature tend to compensate each other when the contributions of $\bar{\nu}_s$ emitted in every direction are added up. In the calculation below we take both semispheres into account when explicitly computing the asymmetry in the total momentum $\mathcal{K}$ emitted by the cooling protoneutron star. In this way, we improve the calculation of Ref. [22] where the temperature within the resonance shell was assumed to be uniform.

The active neutrinos and antineutrinos are emitted isotropically from their respective neutrinospheres and do not contribute to the momentum asymmetry $\Delta\mathcal{K}$. Therefore, the only nonvanishing contribution can come from the $\bar{\nu}_s$. If their emission lasts for an interval Δt of the order of a few seconds, then in the static model we are using for the protostar $\Delta\mathcal{K} = |K_B|\,\Delta t$, where K_B is the component along the magnetic field direction of the neutrino momentum emitted per time unit. According to Eq. (11) we can write

$$K_{B} = \frac{1}{(2\pi)^2} \int_0^\infty k^3 dk \int_0^\pi \sin\theta d\theta \int_0^{\frac{\pi}{2}} \sin\theta' d\theta' \int_0^{2\pi} d\phi'\, r_{R_\ell}^2(\mathbf{k}) f_{\bar{\nu}_s} \hat{\mathbf{k}} \cdot \hat{\mathbf{B}}\,, \quad (58)$$

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where $\hat{\mathbf{k}} \cdot \hat{\mathbf{B}} = \cos\theta\cos\theta' - \sin\theta\sin\theta'\sin\phi'$ and the distribution function for sterile neutrinos $f_{\bar{\nu}_s}$ is evaluated at r_{R_ℓ}. We have chosen a reference frame (x, y, z) fixed to the protostar where the magnetic field coincides with the z-axis, $\mathbf{B} = B\,\hat{\mathbf{z}}$. In addition, at each point within the resonance shell we use the local frame introduced in Section 2 to evaluate the components of the energy-momentum tensor. Here, the upper limit for the integration on θ' is $\pi/2$ instead of π, because we have to include only the contributions from the sterile neutrinos going outside.

To proceed further with the calculation of K_B we need to know $f_{\bar{\nu}_s}$. The proper treatment of the interplay of collisions and oscillations for trapped neutrinos requires the use of the density matrix. For our purpose a less elaborate description will suffice and we put $f_{\bar{\nu}_s} \cong \mathcal{P}(\bar{\nu}_\ell \to \bar{\nu}_s) f_{\bar{\nu}_\ell}$, where $\mathcal{P}(\bar{\nu}_\ell \to \bar{\nu}_s)$ is the probability for $\bar{\nu}_\ell$ to convert into $\bar{\nu}_s$. Let us assume that the adiabaticity condition given by Eq. (57) is verified in the interior of the $\bar{\nu}_\ell$ neutrinosphere, and hence for small mixing $\mathcal{P}(\bar{\nu}_\ell \to \bar{\nu}_s) \cong 1$. Accordingly in Eq. (58) we take $f_{\bar{\nu}_s}(r_R) \simeq f_{\bar{\nu}_\ell}(r_R)$, with

$$
f_{\bar{\nu}_\ell}(r_R) \cong f_{\bar{\nu}_\ell}^{eq}(r_{o_\ell}) - \left(\frac{\cos\theta'}{\varkappa_{\bar{\nu}_\ell} k^2 \rho} \frac{df_{\bar{\nu}_\ell}^{eq}}{dr} - \hat{\mathbf{k}} \cdot \hat{\mathbf{B}} \frac{df_{\bar{\nu}_\ell}^{eq}}{dr} \delta_\ell(k) \right) \Bigg|_{r_{o_\ell}}, \tag{59}
$$

where a term proportional to the derivative of $\rho^{-1} df_{\bar{\nu}_\ell}^{eq}/dr$ has been neglected, in agreement with the diffusive approximation we are employing. Substituting the expansion (59) into Eq. (58) and keeping terms that are at most linear in δ_ℓ we find

$$
\begin{aligned}
K_B \cong \frac{2}{3\pi} \int_0^\infty dk \; k^3 r_{o_\ell}(k)\, \delta_\ell(k) \\
\left(f_{\bar{\nu}_\ell}^{eq} - \frac{1}{2\varkappa_{\bar{\nu}_\ell} k^2 \rho} \frac{df_{\bar{\nu}_\ell}^{eq}}{dr} + \frac{1}{2} r_{o_\ell}(k) \frac{df_{\bar{\nu}_\ell}^{eq}}{dr} \delta_\ell(k) \right) \Bigg|_{r_{o_\ell}}.
\end{aligned} \tag{60}
$$

The energy released during the core collapse of the progenitor star is approximately equipartitioned among all the neutrino and antineutrino flavors. In the context of our analysis this means that $K_r \Delta t \simeq \mathcal{K}/6$, where K_r represents the momentum rate of the sterile neutrinos in the radial direction and is given by an expression similar to Eq. (58) with $\hat{\mathbf{k}} \cdot \hat{\mathbf{B}}$ replaced by $\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}$. Following the same procedure outlined above we find

$$
K_r \cong \frac{1}{2\pi} \int_0^\infty dk \; k^3 \; r_{o_\ell}^2(k) \left(f_{\bar{\nu}_\ell}^{eq} - \frac{2}{3\varkappa_{\bar{\nu}_\ell} \rho k^2} \frac{df_{\bar{\nu}_\ell}^{eq}}{dr} \right) \Bigg|_{r_{o_\ell}}. \tag{61}
$$

In terms of K_B and K_r the fractional asymmetry in the total momentum neutrinos carry away becomes

$$
\frac{\Delta\mathcal{K}}{\mathcal{K}} = \frac{|K_B|}{6 K_r}. \tag{62}
$$

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To evaluate the remaining integrals in Eqs. (61) and (60) the explicit dependence on k of the functions r_{o_ℓ} and $\mathcal{D}_\ell$ have to be known. Simple analytical results can be derived by replacing these functions by the constant quantity $\bar{r}_{o_\ell}$ and $\overline{\mathcal{D}}_\ell = \mathcal{D}_\ell(\bar{r}_{o_\ell})$, where $\bar{r}_{o_\ell}$ is determined from the resonance condition when k is replaced by its thermal average $3.15\,T(r)$:

$$6.3\,T(\bar{r}_{o_\ell})\,b_\ell(\bar{r}_{o_\ell}) = \Delta m^2 \cos 2\theta\,. \tag{63}$$

Proceeding in this manner we get

$$K_r \cong \frac{\pi \bar{r}_{o_\ell}^2}{48} \left(\frac{7\pi^2 T^4}{5} - \frac{4}{3\kappa_{\bar{\nu}_\ell}\rho} \frac{dT^2}{dr} \right)\Bigg|_{\bar{r}_{o_\ell}}, \tag{64}$$

$$K_B \cong \frac{\pi \bar{r}_{o_\ell} \bar{\delta}_\ell}{36} \left(\frac{7\pi^2 T^4}{5} + \frac{7\pi^2 \bar{r}_{o_\ell}}{10} \frac{dT^4}{dr} - \frac{1}{\varkappa_{\bar{\nu}_\ell}\rho} \frac{dT^2}{dr} \right)\Bigg|_{\bar{r}_{o_\ell}}. \tag{65}$$

In these expressions the terms that depend on dT^2/dr come from the diffusive part of the distribution function. In the regime considered here they are smaller than the other contributions and in what follows we will neglect them.

Suppose that the pulsar momentum K_{pul} is entirely due to the active-sterile oscillation driven kick, then $\Delta\mathcal{K} = K_{pul}$. For a pulsar with a mass $M \approx M_\odot$ and a translational velocity $v \approx 500\,\mathrm{km\,s^{-1}}$ we have $K_{pul} \approx 10^{41}\,\mathrm{g\,cm\,s^{-1}}$. Since neutrinos are relativistic, the total amount of momentum they carry is equal to the gravitational energy liberated by means of them, that is $\mathcal{K} \approx 10^{43}\,\mathrm{g\,cm\,s^{-1}}$. Therefore, as mentioned in the Introduction, $\Delta\mathcal{K}/\mathcal{K}$ must be of the order of 0.01 to obtain the required kick. In this case Eq. (62) yields

$$eB \cong \frac{0.045\,\bar{r}_{o_\ell}}{\left| (1 + 2\bar{r}_{o_\ell}\bar{h}_T^{-1})\overline{\mathcal{D}}_\ell \right|}\,, \tag{66}$$

with $\bar{h}_T^{-1}$ denoting the logarithmic derivative of $T(r)$ evaluated at $\bar{r}_{o_\ell}$. This formula allows us to determine the magnitude of the magnetic field once the temperature, the baryon density, and the electron fraction profiles are known. Below we do it for the $\bar{\nu}_\mu$–$\bar{\nu}_s$ conversion under the assumption that the resonance layer is entirely within the $\bar{\nu}_\mu$-sphere. However, we will first indicate how $Y_e(r)$ is determined by means of $T(r)$ and $\rho(r)$ within our approximate description of a protoneutron star.

For $\mu_{\nu_e} \simeq 0$, the β equilibrium gives the relation $\mu_n - \mu_p \simeq \mu_e$ among the chemical potentials of the background fermions. Hence, for non-relativistic nucleons $N_n/N_p \simeq \exp[-(m_n - m_p - \mu_e)/T]$ and

$$Y_e \simeq \frac{1}{1 + e^{\mu_e/T}}\,, \tag{67}$$

where we used the fact that in the condition prevailing in a protoneutron star μ_e is much larger than the difference between the neutron and the proton masses.

Since electrons are degenerated $\mu_e = \left(3\pi^2 \rho Y_e / m_n\right)^{1/3}$ and Eq. (67) is an implicit equation for Y_e to be solved numerically once ρ and T are known as a function of r. Taking the derivative of Eq. (67) with respect to r, $h_{Y_e}^{-1}$ can be expressed in terms of h_ρ^{-1} and h_T^{-1} as follows

$$h_{Y_e}^{-1} = -\left(h_\rho^{-1} - 3h_T^{-1}\right) \frac{1 - Y_e}{1 - Y_e + \frac{3T}{\mu_e}} \,. \tag{68}$$

To estimate B we now assume that the density and the temperature are related according to $\rho = \rho_c \left(T/T_c\right)^3$[33], where ρ_c and T_c are the respective values of these quantities at the core radius R_c. From this expression we see that $h_\rho^{-1} = 3\,h_T^{-1}$ which implies that $h_{Y_e}^{-1} = 0$ (see Eq. (68)). Therefore, the electron fraction Y_e and as a consequence the ratio μ_e/T are constants. As core parameters we take $R_c = 10\,\mathrm{km}$, $\rho_c = 10^{14}\,\mathrm{g\,cm^{-3}}$, and $T_c = 40\,\mathrm{MeV}$, while for the density profile we adopt the potential law $\rho(r) = \rho_c \left(R_c/r\right)^4$ which yields $h_\rho^{-1} = -4/r$ and $T(r) = T_c \left(R_c/r\right)^{4/3}$. The $\bar{\nu}_\mu$-sphere is taken stationary at a density of about $10^{12}\,\mathrm{g\,cm^{-3}}$ that corresponds to a radius $R_{\bar{\nu}_\mu} \simeq 3\,R_c$ and a temperature $T_{R_{\bar{\nu}_\mu}} \simeq 9\,\mathrm{MeV}$. Using these results as well as the formula for $\mathcal{D}_\mu$ given in Eq. (56), from Eq. (66) we obtain the following simple result:

$$B = 5 \times 10^{17} \left(1 + 0.3\frac{T_c}{\bar{T}}\right)^{-1} \frac{\bar{T}}{T_c}\,\mathrm{G}\,, \tag{69}$$

where $\bar{T} = T(\bar{r}_{o_\mu})$, with $R_c \leq \bar{r}_{o_\mu} \leq R_{\bar{\nu}_\mu}$. We see that the required intensity of the magnetic field decreases monotonically from $3.8 \times 10^{17}\,\mathrm{G}$ at the core radius to $4.8 \times 10^{16}\,\mathrm{G}$ at the $\bar{\nu}_\mu$-sphere. According to Eqs. (52) and (47), within the specified interval for $\bar{r}_{o_\mu}$ the range of the allowed values of the oscillation parameters are $7 \times 10^8\,\mathrm{eV^2} \gtrsim \Delta m^2 \cos 2\theta \gtrsim 10^5\,\mathrm{eV^2}$, and for small mixing we have $25\,\mathrm{KeV} \gtrsim m_s \gtrsim 0.3\,\mathrm{KeV}$. The requirement that the resonant conversion be adiabatic imposes the condition $\tan^2 2\theta \gtrsim 3 \times 10^{-11} \left(\bar{r}_{o_\mu}/R_c\right)^3$ on the mixing angle. A last comment is in order, the ratio of the diffusive term to the isotropic one in Eqs. (64) and (65) can be expressed as $10^{-4} \left(\bar{r}_{o_\mu}/R_c\right)^{17/3}$, which illustrates the validity of the approximation done in writing Eq. (66). In the next section we repeat these calculations in the realm of the spherical Eddington model, which provides a simple but quite physically reasonable description of a static neutrino atmosphere.

6   Spherical Eddington Model

The actual configuration of the neutrinosphere and the resonance region depend on details of the protoneutron star, such as the density, temperature, pressure, and leptonic fraction profiles. The asymmetry in the momentum flux also depends on these features and on the configuration of the magnetic field. The present knowledge about protoneutron stars is not enough to single out a definite model for their structure, but there are some well established general

characteristics that help towards proposing models that are both workable and plausible. An important feature is that the size and the shape of the different profiles do not suffer sudden changes during the time interval when the main neutrino emission takes place, i.e. $0.5\,s \lesssim t \lesssim 10\,s$. Thus, as a first approximation we can consider a stationary regime that greatly simplifies calculations. Another characteristic to be taken into account is that the bulk of neutrinos is produced in the inner core. Consequently, we assume that there is no neutrino production outside the core and hence the flux of diffusing neutrinos is conserved in this region. This flux is responsible for the transport of the energy liberated by the protostar.

As mentioned before, the system is supposed to be constituted by nucleons, with a density ρ, electrons, neutrinos and antineutrinos. Neutrinos are in thermal equilibrium with nucleons, satisfying a transport regime consistent with the diffusion approximation, and are assumed to have a vanishing chemical potential, $\mu_\nu = \mu_{\bar\nu} = 0$. The total energy density, the radial energy flux, and the pressure for neutrinos and antineutrinos in the case of spherical symmetry, can be obtained from Eqs. (24), (25) and (10)

$$U = \sum_\ell U_{\nu_\ell} = \frac{7}{40}\pi^2 T^4, \tag{70}$$

$$F = \sum_\ell F_{\nu_\ell} = -\frac{1}{36\bar\varkappa\rho}\frac{dT^2}{dr}\ , \tag{71}$$

$$P_\nu = \sum_\ell T_{\nu_\ell}^{ii} = \frac{U}{3}\ , \tag{72}$$

where $\bar\varkappa^{-1} = \varkappa_{\nu_e}^{-1} + 2\varkappa_{\nu_{\mu,\tau}}^{-1}$. The main contributions to the neutrino opacities were discussed in Section III. Using the results given there for $\varkappa_{\nu_e}$ and $\varkappa_{\nu_{\mu,\tau}}$ (Eqs. (21) and (22)) we obtain $\bar\varkappa = 3\times 10^{-26}$ MeV^{-5}.

If in addition we assume that baryons constitute a nonrelativistic ideal gas, we have the Eddington model. It is simple and physically well justified, allowing a detailed discussion of the relevant characteristics of the protostar and the geometry of the resonance region. Originally proposed to describe a stellar photosphere, this model was adapted by Schinder and Shapiro[48] to a neutrino atmosphere with a plane geometry. The more realistic case of a spherical atmosphere was considered in Ref. [19] to analyze the geometrical effect on the asymmetric neutrino emission induced by resonant oscillations. Photons and electrons are of course present and, in fact, electrons make the leading contribution to the effective potential in the case of oscillations among active neutrinos. However, we can ignore both photons and electrons for the hydrodynamical description of the system.

The state equation that defines the system is given by the sum of the contributions from nucleons and neutrinos to the pressure gas, which is

$$P = \frac{\rho T}{m_n} + \frac{U}{3}. \tag{73}$$

In the Newtonian limit for the metric ($g_{oo} = -1 - 2\phi$, $g_{oj} = 0$, and $g_{ij} = \delta_{ij}$, with $\phi = -GM(r)/r$) the conservation of the total (neutrinos + matter) energy-momentum tensor yields the energy flux conservation

$$\frac{\partial(r^2 F)}{\partial r} = 0 \tag{74}$$

and the hydrostatic equilibrium equation

$$\frac{dP}{dr} = -(P + \rho + U)\frac{GM}{r^2}. \tag{75}$$

Here, $M(r) = 4\pi \int_0^r dr' \, r'^2 \rho(r')$ is the mass enclosed up to a distance r from the center.

Through the atmosphere the baryon density is $\rho \simeq (10^{11} - 10^{14})\,\mathrm{g\,cm^{-3}}$ and $T \simeq (4 - 40)\,\mathrm{MeV}$, and thus we have $U \simeq (10^{-3} - 10^{-2})\rho$ and $P \simeq (4 \times 10^{-3} - 4 \times 10^{-2})\rho$. This means that on the right-hand side of Eqs. (73) and (75) we can ignore the contributions coming from the energy density of neutrinos and the baryonic density. In addition, Eq. (74) implies that $F = L_c/4\pi r^2$, where L_c is the luminosity of the protostar. Taking all the above into account we arrive at the following set of equations for an isotropic atmosphere of neutrinos in thermal equilibrium with an ideal gas of nucleons:

$$\frac{dT^2}{dr} = -\rho\,\frac{9\bar{\varkappa}\,L_c}{\pi r^2}, \tag{76}$$

$$P = \frac{\rho}{m_n}\,T, \tag{77}$$

$$\frac{dP}{dr} = -\rho\,\frac{GM}{r^2}, \tag{78}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho. \tag{79}$$

Solving this system of equations yields the profiles of four functions throughout the atmosphere: pressure $P(r)$, temperature $T(r)$, baryonic density $\rho(r)$, and the enclosed mass $M(r)$. From them any other functions of interest can be calculated. To analyze the system it is convenient to introduce adimensional variables normalized to the corresponding values at the core: $x = r/R_c$, $m(x) = M/M_c$, $t(x) = T/T_c$, $\varrho(x) = \rho/\rho_c$, and $p(x) = P/P_c$. Proceeding in this way, Eqs. (76)-(79) can be rewritten

$$\frac{dt}{dx} = -b_c\frac{\varrho}{tx^2}, \tag{80}$$

$$p = t\varrho, \tag{81}$$

$$\frac{dp}{dx} = -c_c\frac{m\varrho}{x^2}, \tag{82}$$

$$\frac{dm}{dx} = d_c x^2 \varrho, \tag{83}$$

where
$$b_c = \frac{9\bar{\varkappa}\rho_c L_c}{2\pi R_c T_c^2}, \qquad c_c = \frac{Gm_n M_c}{R_c T_c}, \qquad d_c = \frac{4\pi\rho_c R_c^3}{M_c}. \tag{84}$$

are adimensional constants. Thus the set of functions that solve the previous system of equations depends only on these three combinations of the core parameters.

By eliminating ϱ in Eqs. (80) and (82) we arrive to

$$\frac{dp}{dx} = \frac{c_c}{b_c} mt \frac{dt}{dx}. \tag{85}$$

The last equation is easily integrated as follows:

$$p = \frac{t^2 - a}{1 - a}, \tag{86}$$

where

$$a(x) = 1 - \frac{2b_c}{c_c \bar{m}(x)} \tag{87}$$

and $\bar{m}(x)$ is an effective mass defined by

$$\int_1^x dx'\, m(x')\, t\, \frac{dt}{dx'} \equiv \bar{m}(x) \int_1^x dx'\, t\, \frac{dt}{dx'}. \tag{88}$$

From Eqs. (81) and (86) we can also express the density in terms of the temperature as

$$\varrho = \frac{t^2 - a}{t(1 - a)}, \tag{89}$$

which, together with Eq. (80), yields a first order differential equation for t

$$\frac{dt}{dx} + \frac{b_c}{t^2 x^2} \frac{t^2 - a}{1 - a} = 0. \tag{90}$$

At $x = 1$ ($R = R_c$) we have $dt/dx|_{x=1} = -b_c$, which is independent of $a(x)$. For an idealized atmosphere, where the model would apply to the whole space, the physical solutions would correspond to an infinite protostar where the temperature has an asymptotic behavior for $x \gg 1$, such that the adimensional temperature tends to $t_s \simeq \sqrt{a}$. Thus, the function $a(x)$ varies in the range $1 - 2b_c/c_c < a < t_s^2$ for $1 < x < \infty$.

Eq. (90) has no analytical solution when a is a function of x. An approximate solution can be found by the following procedure. Let us consider the differential equation (90) with a constant. Then, an analytical (implicit) solution is given by

$$t - 1 + \frac{\sqrt{a}}{2}\left[\ln\left(\frac{t - \sqrt{a}}{t + \sqrt{a}}\right) - \ln\left(\frac{1 - \sqrt{a}}{1 + \sqrt{a}}\right)\right] = \frac{b_c}{1 - a}\left(\frac{1}{x} - 1\right). \tag{91}$$

with $1 > t > \sqrt{a}$. If we replace the constant a by a well behaved function of x, the above expression still satisfies Eq. (90) at $x = 1$. For an infinite atmosphere,

a good approximation to the exact solution is given by Eq. (91), with a now a function of $t(x)$:

$$a(t) = 1 - \frac{2b_c}{c_c} - A(1 - t) , \qquad (92)$$

where $A = \left(1 - \frac{2b_c}{c_c} - t_s^2\right)/(1 - t_s)$. From Eq. (92), we see that $a(t_s) = t_s^2$ and $a(1) = 1 - 2b_c/c_c$, which means that this ansatz fits the extreme values of $a(x)$.

Once ρ and T are known, we can find the electron fraction Y_e as a function of r by solving the implicit expression derived in Eq. (67). To examine the prediction of the model in connection with the kick mechanism, we consider a protoneutron star with reasonable values for its core parameters, the same as in Ref. [19]. Thus we take $M_c = M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$, $R_c = 10\,\mathrm{km}$, $L_c = 2 \times 10^{52}\,\mathrm{erg\,s^{-1}}$, $\rho_c = 10^{14}\,\mathrm{g\,cm^{-3}}$, and $T_c = 40\,\mathrm{MeV}$. In this case,

$$b_c = 1.96 , \qquad c_c = 3.52 , \qquad d_c = 0.625 . \qquad (93)$$

A numerical estimation for the asymptotic value of the solution of Eqs. (80)-(83) gives $t_s = 0.12$. The electron fraction takes the value $Y_e = 0.08$ at the core.

According to Eq. (20) in terms of the adimensional functions the mean free path for the electron neutrino becomes

$$\lambda_{\nu_e} \simeq \frac{8}{\varrho\, t^2} \ \mathrm{cm} . \qquad (94)$$

Inserting the above expression in the integrand of Eq. (1) and evaluating the resulting integral numerically we obtain $x_{\nu_e} \simeq 2.6$ for the radius of the ν_e-sphere. This yields $\rho(R_{\nu_e}) \simeq 1.3 \times 10^{11}\,\mathrm{g\,cm^{-3}}$, $T(R_{\nu_e}) \simeq T_s = 4.8\,\mathrm{MeV}$, and $Y_e(R_{\nu_e}) \simeq 0.1$, which are in reasonable agreement with typical values from numerical simulations[32, 4]. In the same way, for the muon (tau) neutrino we get $x_{\nu_{\mu,\tau}} \simeq 2.2$. The profiles that correspond to the solution of this model are given in Fig. 1.

From Eq. (66) we can estimate again the intensity of the magnetic field required for the kick in the case of $\bar\nu_\mu \longleftrightarrow \bar\nu_s$. Besides the temperature, density, and electron fraction profiles we also need their logarithmic derivatives. For T and ρ they are immediately calculated from Eqs. (80)-(82), and the corresponding results are

$$h_T^{-1} = -b_c \frac{\varrho}{tx} \frac{1}{r} , \qquad (95)$$

$$h_\rho^{-1} + h_T^{-1} = -c_c \frac{m}{tx} \frac{1}{r} . \qquad (96)$$

The formula for $h_{Y_e}^{-1}$ is obtained substituting the above expressions into Eq. (68). Now we have all the quantities needed to determine the magnetic field from Eq. (66) in the spherical Eddington model. The result is plotted in Fig. 2. From the curve presented in this figure we see that B decreases from a value of $8 \times 10^{16}\,G$ in the core up to a minimum of $3.8 \times 10^{16}\,G$ at $r_{o_\mu} \simeq 16\,\mathrm{km}$. From this point the magnetic field increases steadily as we approach the surface of

the neutrinosphere. In contrast with the model used at the end of Section 5, here at $r \simeq 21 \; km$ the contributions from the geometrical and the temperature variations compensate each other. At this radius the factor $(1 + 2\bar{r}_{o_\ell})$ in the denominator of Eq. (66) vanishes and the magnetic field takes arbitrarily large values. In other words, no kick can be generated by this mechanism in the regions near the surface of the ν_μ-sphere.

In the region between the core and the surface of the neutrinosphere $R_c \leq r_{o_\mu} \leq R_{\nu_\mu}$, the range of values allowed for the oscillation parameters is

$$10^9 \text{eV}^2 \gtrsim \Delta \text{m}^2 \cos 2\theta \gtrsim 10^6 \text{eV}^2, \tag{97}$$

For a small mixing angle this inequality translates into the condition $30 \, \text{KeV} \gtrsim \text{m}_s \gtrsim 1 \, \text{KeV}$ for the sterile neutrino mass. Small mixing is allowed by the adiabaticity condition, which warrants an efficient flavor neutrino conversion for $\tan^2 2\theta > 7 \times 10^{-12}$ at $r_{o_\mu} = R_c$ and $\tan^2 2\theta > 3 \times 10^{-9}$ at $r_{o_\mu} = R_{\nu_\mu}$. The results for m_s and the mixing angle agree with those derived in the previous section and are compatible with a sterile neutrino that is a viable dark matter candidate[49].

7   Conclusion

In this work we have examined in detail a possible explanation of the large drift velocities observed in pulsars in terms of an asymmetric neutrino emission. The asymmetry is a consequence of the resonant neutrino conversions affected by the strong magnetic field characteristic of protoneutron stars. We have shown that for active-sterile neutrino oscillations this is a feasible kick mechanism. Two conditions must be simultaneously satisfied to generate a natal kick by this mechanism. First, the conversion has to take place at the interior of the neutrinosphere of the active neutrino, and second the magnetic field has to be intense enough in order to induce a momentum asymmetry of the required magnitude.

We have made detailed calculations by means of the neutrino distribution function in the diffusion approximation, combined with the idea of a spherical resonance shell for neutrinos with momentum k that move in different directions relative to $\mathbf{B}$. This scheme provides a better description of the problem than the one formulated in terms of a single deformed sphere acting as an effective emission surface of sterile neutrinos. In our approach the sterile neutrinos produced in the resonance shell move in every direction relative to $\mathbf{B}$. However, those going toward the interior of the protoneutron star cross again the resonance region and are reconverted into active neutrinos. These are within their own neutrinosphere and become thermalized. As a consequence, only the outgoing sterile neutrinos contribute to the total momentum emitted by the protostar.

Two opposite effects have to be taken into account when computing the fractional momentum asymmetry. One is purely geometric, and comes from the difference in the areas of the semispherical emission surface for neutrinos moving in opposite directions with the same momentum magnitude. The other is due to

the radial temperature gradient, by which sterile neutrinos produced at different depths have unequal energy. In general both contributions do not compensate and thus we obtain a non null fractional asymmetry in the total momentum. Explicit results have been obtained for two simple models of a protostar, with the matter background assumed to be composed by nonrelativistic nucleons and degenerated electrons. One of them, which we worked with in detail, is the Eddington model for a spherical neutrino atmosphere.

Magnetic fields of the order of $10^{16} - 10^{17}$ G are needed to reproduce the observed pulsar velocities. At first sight these fields could seem rather large, compared with the estimated values at the surface of a protoneutron star ($B_s \sim 10^{13}$ G), but in fact intensities as high as $B_c \sim 10^{18}$ G are possible at the core. A given parametrization for the magnetic field at the interior of a protoneutron star is[36]

$$B = B_s + B_c \left[1 - e^{-\beta(\rho/\rho_s)^\gamma} \right],$$

(98)

with $\beta \simeq 10^{-5}$ and $\gamma \simeq 3$. By adopting this profile as an upper boundary for the magnetic field in the protostar, we can see that the required field remains well below this boundary in most of the region within the neutrinosphere. In addition, the oscillation parameters are compatible with the allowed region for sterile neutrinos to be warm dark matter, leaving the mechanism as an attractive possibility to explain the proper motion of pulsars.

8   Acknowledgments

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Figure captions

FIG. 1: The different profiles normalized to the corresponding value at the core for the Eddington model discussed in Section VI.

FIG. 2: The magnetic field B required to produce the kick by the $\bar{\nu}_\mu \rightarrow \bar{\nu}_s$ resonant conversion for the Eddington model discussed in Section VI, normalized to the field required at the core, B_c.